Dynamics of US Buffett Indicator [*]

Hiro Asano

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Points of this study

1 The W5K and GDP are both nonstationary as they are known, but cointegrated.

2 The Buffett indicator is a mean-reverting Gaussian process with a Poisson jump process.

3 The reversion mean of the Buffett indicator is increasing along with GDP, as is often pointed out.

4 However, the increasing rate of the W5K equals the GDP growth rate, as the Buffett indicator implies.

5 This study detects four outliers in the W5K, which coincided with four negative shocks to the US economy.

6 The ECM of the Buffett indicator is variance stationary as opposed to the EMH.

This study investigates the dynamic relationship between the U.S. economy's goods and stock markets. It uses the market capitalization of the Wilshire 5000 index (W5K) and Gross Domestic Product (GDP) as size measures for those markets. The W5K represents the aggregate market capitalization of all publicly traded firms, and GDP represents the value of all goods and services domestically produced. The W5K and GDP both exhibit dynamics of long-term trends and short-term fluctuations.

The study assumes an underlying relationship between the W5K and GDP. Value added is defined as sales revenue less the costs of intermediate goods such as materials and energy. The profit equals value added less labor costs and less depreciation. These items are proportional to the output to some extent. In particular, value-added and profits are roughly proportional. At the same time, investors evaluate a firm based on its profit and holding assets, and contemplate its stock price. Thus, a firm's profit should reflect on its market capitalization, which is the product of its stock price and the number of its outstanding shares.

Profits are a part of value added. If the discount rate and annual profits are constant over the years, a stock price theoretically equals yearly profits divided by the discount rate. In other words, the discount rate's inverse is the theoretical value of the price-earnings ratio (P/E ratio). The W5K is the aggregate of all firms' market capitalization, while GDP is the aggregate value-added of all firms operating in a country. In this way, the W5K and GDP are related to each other.

The efficient-market hypothesis (EMH)[*1] implies the random-walk hypothesis that the stock price is a random-walk process.[*2] After a large number of steps, a random walk converges toward a Wiener process.[*3] As a result, the return on stocks should follow a normal distribution, i.e., Gaussian noise. Also, the EMH implies that the current return on stocks is independent of the past returns on stocks. In addition, if the return on stocks behaves consistently over time, the return on stocks follows an independent and identically distributed

[*] This is a preliminary and incomplete draft. The author is still working on many issues, and suggestions are welcome. (Email: asanohiro559@gmail.com)

[*1] *Wikipedia*, *The Free Encyclopedia*, s.v. "Efficient-market hypothesis," (accessed February 5, 2025), `https://en.wikipedia.org/wiki/Efficient-market_hypothesis`

[*2] *Wikipedia*, *The Free Encyclopedia*, s.v. "Random walk," (accessed March 12, 2025), `https://en.wikipedia.org/wiki/Random_walk`

[*3] *Wikipedia*, *The Free Encyclopedia*, s.v. "Wiener process," (accessed March 12, 2025), `https://en.wikipedia.org/wiki/Wiener_process`

(i.i.d.) normal distribution, and is white Gaussian noise.[*4]

A stock price is often said to exhibit regression toward the mean,[*5] or to be a mean-reverting process, also known as the Urnstein-Uhlenbeck process.[*6] If it is the case, it contradicts the EMH. The EMH implies that the return on stocks is the Wiener process whose drift term is constant, while the drift term is dependent on the current value of the process for the mean-reverting process.

The study analyzes the dynamic relationship between the W5K and GDP by time series econometrics. The W5K and GDP are both known as nonstationary or unit-root processes.[*7] When nonstationary time series data is regressed on another nonstationary time series, the regression can be spurious or nonsensical. On the other hand, when both are related, nonstationary time series data could be cointegrated[*8] and regression with cointegrated data is possibly superconsistent.[*9]

The study employs ordinary least squares (OLS) estimations of the error correction model (ECM).[*10] It examines whether the W5K and GDP are cointegrated, and whether OLS residuals are white Gaussian noise. It concludes that they are cointegrated, and, after removing outliers from the dataset, the ECM residuals are Gaussian but not white. Here, the Buffett indicator plays a key role. [*11][*12] The study also investigates whether the Buffett indicator is a mean-reverting process or whether the indicator shows the regression toward the mean. If the indicator is the mean-reverting process, it tends to return to the long-run equilibrium level. Finally, given GDP, the study simulates both the Buffett indicator and the W5K.

1 Data

This study analyses the US market capitalization and Gross Domestic Product (GDP). The former employs the market capitalization of the Wilshire 5000 index (W5K), which measures the US stock markets, while the latter measures the US economy. The W5K is the aggregate of the market capitalization of all publicly traded US firms. The index was first published on December 31, 1970. The data is published daily, and the study examines the index's quarterly mean. GDP is the aggregate of value added produced domestically. The US government started publishing the US GDP data in the first quarter of 1947. The US GDP data is published quarterly.[*13]

1.1 Data in absolute form

The upper panel of Figure 1 shows the time-series plot of the W5K and GDP. Both the W5K and GDP show long-term trends as well as short-term fluctuations. They have been increasing in the long term, while the W5K has fluctuated more vigorously than GDP in the short term. Also, the panel shows that the W5K has risen

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[*4] *Wikipedia*, *The Free Encyclopedia*, s.v. "White noise," (accessed February 5, 2025), `https://en.wikipedia.org/wiki/White_noise`

[*5] *Wikipedia*, *The Free Encyclopedia*, s.v. "Regression toward the mean," (accessed March 12, 2025), `https://en.wikipedia.org/wiki/Regression_toward_the_mean`

[*6] *Wikipedia*, *The Free Encyclopedia*, s.v. "Ornstein – Uhlenbeck process," (accessed March 12, 2025), `https://en.wikipedia.org/wiki/Ornstein-Uhlenbeck_process`

[*7] *Wikipedia*, *The Free Encyclopedia*, s.v. "Unit root," (accessed March 12, 2025), `https://en.wikipedia.org/wiki/Unit_root`

[*8] *Wikipedia*, *The Free Encyclopedia*, s.v. "Cointegration," (accessed January 5, 2025), `https://en.wikipedia.org/wiki/Cointegration`

[*9] Stock, 1987

[*10] *Wikipedia*, *The Free Encyclopedia*, s.v. "Error correction model," (accessed January 5, 2025), `https://en.wikipedia.org/wiki/Error_correction_model`

[*11] Buffett and Loomis, 2001

[*12] *Wikipedia*, *The Free Encyclopedia*, s.v. "Buffett indicator," (accessed January 5, 2025), `https://en.wikipedia.org/wiki/Buffett_indicator`

[*13] U.S. Bereau of Economic Analysis, Gross Domestic Product [GDP], retrieved from FRED, Federal Reserve Bank of St. Louis; (accessed March 31, 2025), `https://fred.stlouisfed.org/series/GDP`

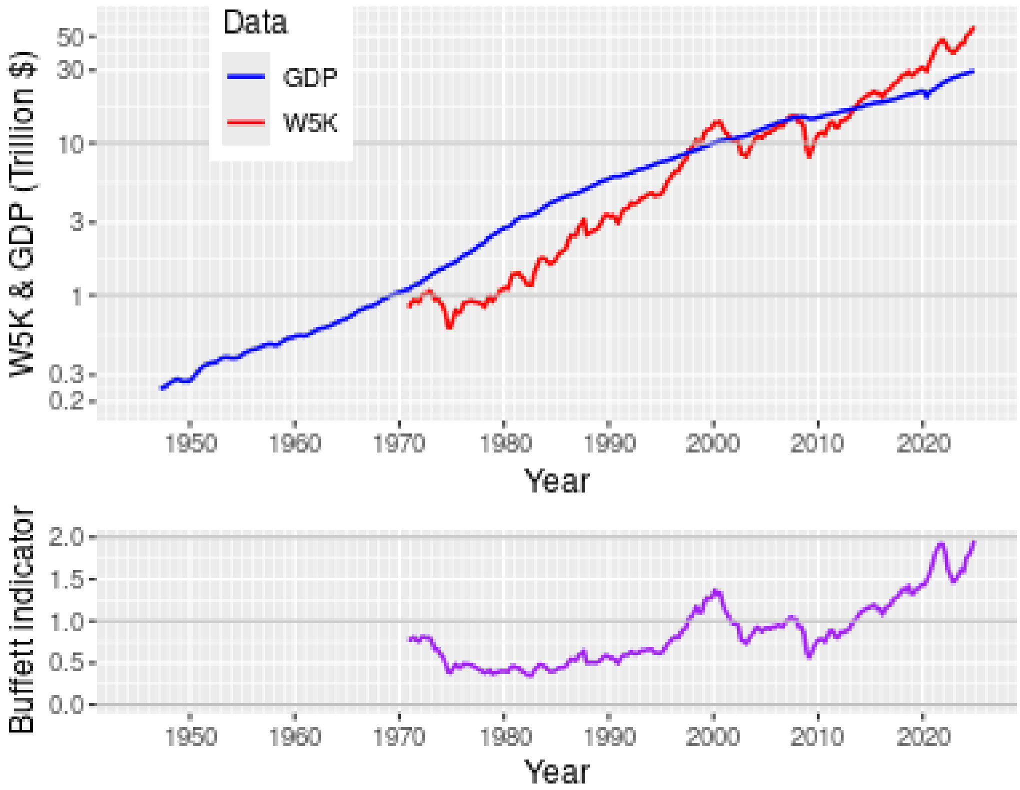

図 1　Time-series plots of W5K, GDP, and Buffett indicator

faster than GDP. The lower panel of Figure 1 shows the time-series plot of the Buffett indicator. The indicator has also fluctuated in the short term and increased in the long term. Its upward trend results from the W5K rising faster than GDP. Its value was about unity around the year 2000.

In the study, the mathematical symbols of the W5K and GDP are, respectively, S and Y, which are natural-logged. The Buffett indicator (BI) is defined as the ratio of the W5K to GDP.

$$BI_t \equiv \frac{S_t}{Y_t} \quad \text{or} \quad lnBI_t \equiv lnS_t - lnY_t \tag{1}$$

where subscript t is a time index. Its value is supposed to equal unity. When the stock price index is either high or low, the Buffett indicator is also high or low. The study investigates the relation between lnS_t and lnY_t. The relation of the Buffett indicator is $lnS_t - lnY_t$. However, to incorporate the upward trend of the Buffett indicator, the study modifies the indicator to $lnS_t - \psi \cdot lnY_t$ with $\psi \approx 1$.

The W5K and GDP are known to be nonstationary or to have unit roots. When one nonstationary series is regressed on another nonstationary time series, the regression could be spurious, i.e., it could show seemingly good results even though the data are irrelevant. The study follows the Box-Jenkins methodology and investigates the nonstationarity of both datasets by using the augmented Dickey-Fuller (ADF) test for nonstationarity. The Box-Jenkins methodology is a four-step procedure.

Step 1 Transform data to make time-series data stationary, often taking differences.

Step 2 Determine the order of an autoregressive moving-average (ARMA) model.

Step 3 Estimate the parameters of the ARMA process.

Step 4 Perform diagnostic analysis.

表 1　Correlation patterns of ARMA process

Process	ACF	PACF
AR(p)	Infinite: damping out	Finite: cuts off after lag p
MA(q)	Finite: cuts off after lag q	Infinite: damping out
ARMA	Infinite: damping out	Infinite: damping out

表 2　ADF test of W5K and GDP

Variable	Test statistic	p value	H_0	T
lnS_t	-2.689718	0.2866228	nonstationarity	217
lnY_t	-2.9818798	0.164046	nonstationarity	217
$lnBI_t$	-3.4836235	0.0453607	nonstationarity	217

Table 1 shows patterns of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) for AR(p), MA(q), and ARMA processes. Figure 2 shows the time-series plots and the correlograms of the W5K, GDP, and the Buffett indicator after 1971. Its first row shows the time-series plot of lnS_t. Its second row shows the correlogram of lnS_t, which shows the persisting ACF, but the PACF is insignificant after the first lag. Therefore, the W5K should be an AR(1) process, and likely a unit root process. Its third row shows the time-series plot of lnY_t and its fourth row shows the correlogram of lnY_t. GDP's correlogram shows the persisting ACF, while the PACF is insignificant after the first lag. Similarly to the W5K, GDP should also be an AR(1) process, and likely a unit root process. Its fifth row shows the time-series plot of $lnBI_t$ and its sixth row shows the correlogram of $lnBI_t$. The Buffett indicator's correlogram shows the persisting ACF, while the PACF is insignificant after the first lag. Like the W5K and GDP, the Buffett indicator should also be an AR(1) process, and possibly a unit root process.

Table 2 shows the results from the ADF tests of the data. Both the W5K and GDP fail to reject the null hypothesis of nonstationarity. However, the Buffett indicator shows a somewhat ambiguous result. The indicator's p value for the ADF test is 4.5360678%.

1.2 Data in first-differenced form

However, after all time-series data are first-differenced, their ACFs become insignificant, similar to their PACFs. The EMH implies that a stock price is a random walk. Then, first-differencing a stock price index such as the W5K supposedly yields white Gaussian noise, or ΔlnS_t should be white Gaussian noise. The white noise means that noise follows an i.i.d. distribution, and the Gaussian noise means that noise is normally distributed. The study employs the Ljung-Box test for independence and the Shapiro-Wilk test for normality. The Ljung-Box test is actually a test whose null hypothesis is no autocorrelation. No autocorrelation is the necessary, but not the sufficient, condition for independence. The study applies these hypothesis tests not only to ΔlnS_t but also to ΔlnY_t and $\Delta lnBI_t$.

The upper panel of Figure 3 is the time-series plot of the first-differenced W5K, ΔlnS_t. Here, ΔlnS_t approximates the retrun on the W5K. The points would not show a long-term trend, or could be constant over time. There are possibly four downward outliers and one upward outlier. The second row of Figure 3 is the correlogram of ΔlnS_t. The first lags of the ACF and the PACF are significant, but other values are insignificant. The left panel of the third row is the histogram of ΔlnS_t, which seems normal. The right panel of the third row is the QQ plot of ΔlnS_t. Most scores follow closely the normal-distribution line. However, there are possibly five outliers: four at the lower end, and one at the upper end.

Table 3 shows the descriptive statistics and the hypothesis tests of the first-differenced W5K, ΔlnS_t. The

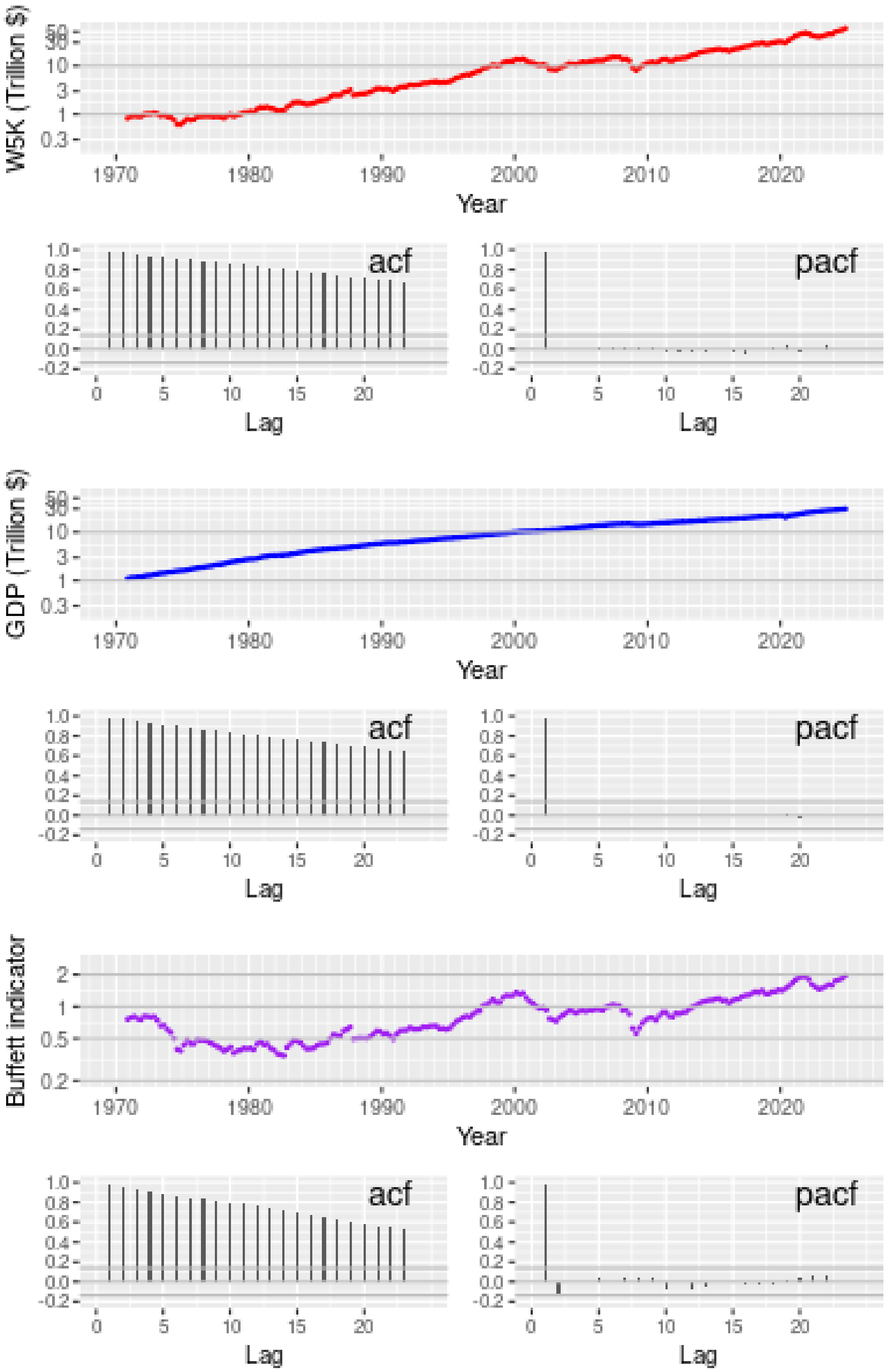

図 2　Time-series plots and correlograms of W5K, GDP, and Buffett indicator

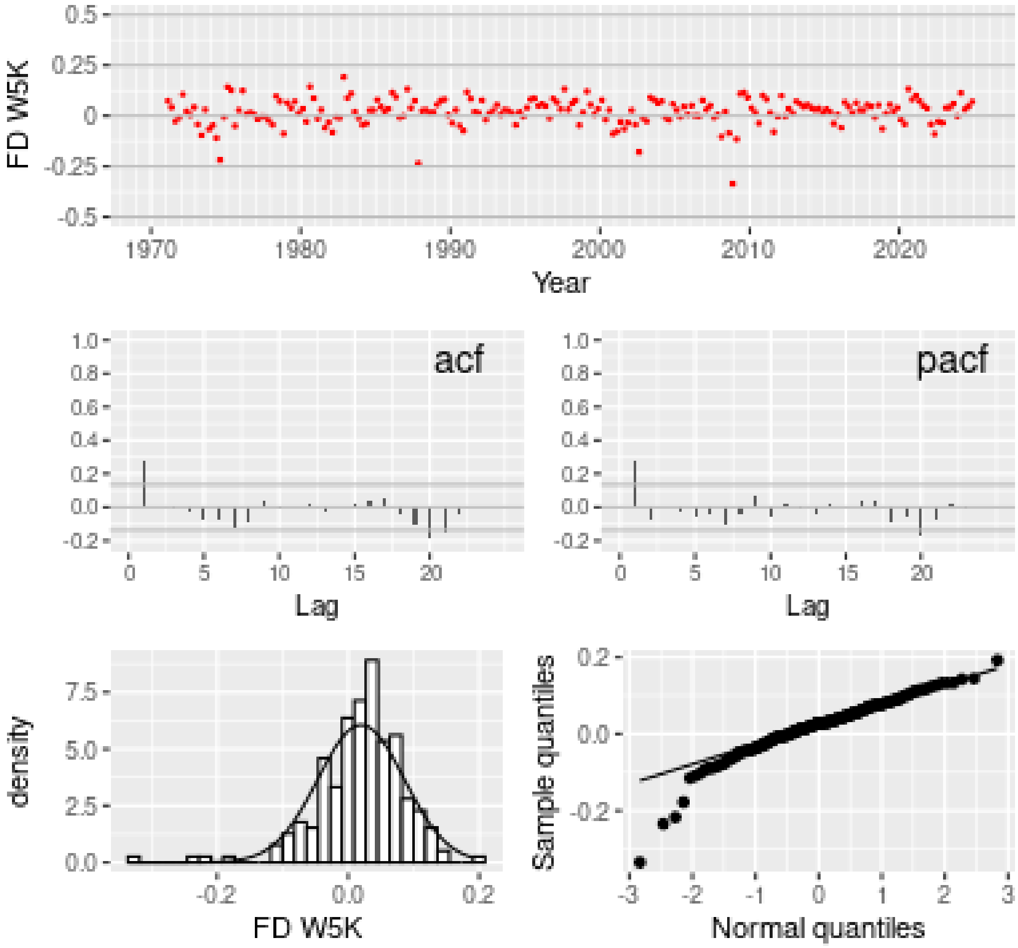

図 3　Time-series plot, correlogram, histogram, and QQ plot of first-differenced W5K

表 3　Descriptive statistics and hypothesis tests of first-diffenced W5K

Mean	Variance	Skewness	Kurtosis
0.0196816	0.0043398	-1.2339941	7.5777916

Test	Test statistic	p value	H_0
ADF test	-6.2204023	< 0.01	nonstationarity
Ljung-Box test	42.2778614 (df = 24)	0.0120103	no autocorrelation
Shapiro-Wilk test	0.9329822	2.2113822×10^{-8}	normality

($T = 216$)

skewness is negative, i.e., a longer tail on the left side, and the kurtosis exceeds three, i.e., leptokurtic. The ADF test rejects the null hypothesis of nonstationarity. In other words, the first-differenced W5K is stationary. The Ljung-Box test fails to reject the null hypothesis of no autocorrelation at the 1% confidence level. The Shapiro-Wilk test rejects the null hypothesis of normality due to negatively-skewed and leptokurtic distribution. Thus, the first-differenced W5K could be white but not Gaussian.

The upper panel of Figure 4 is the time-series plot of first-differenced GDP, ΔlnY_t. The range of ΔlnY_t is narrower than that of ΔlnS_t. First-differenced GDP would have a slightly downward trend, and its range becomes somewhat narrower over time. There are two outliers, one downward and another upward, in 2020.

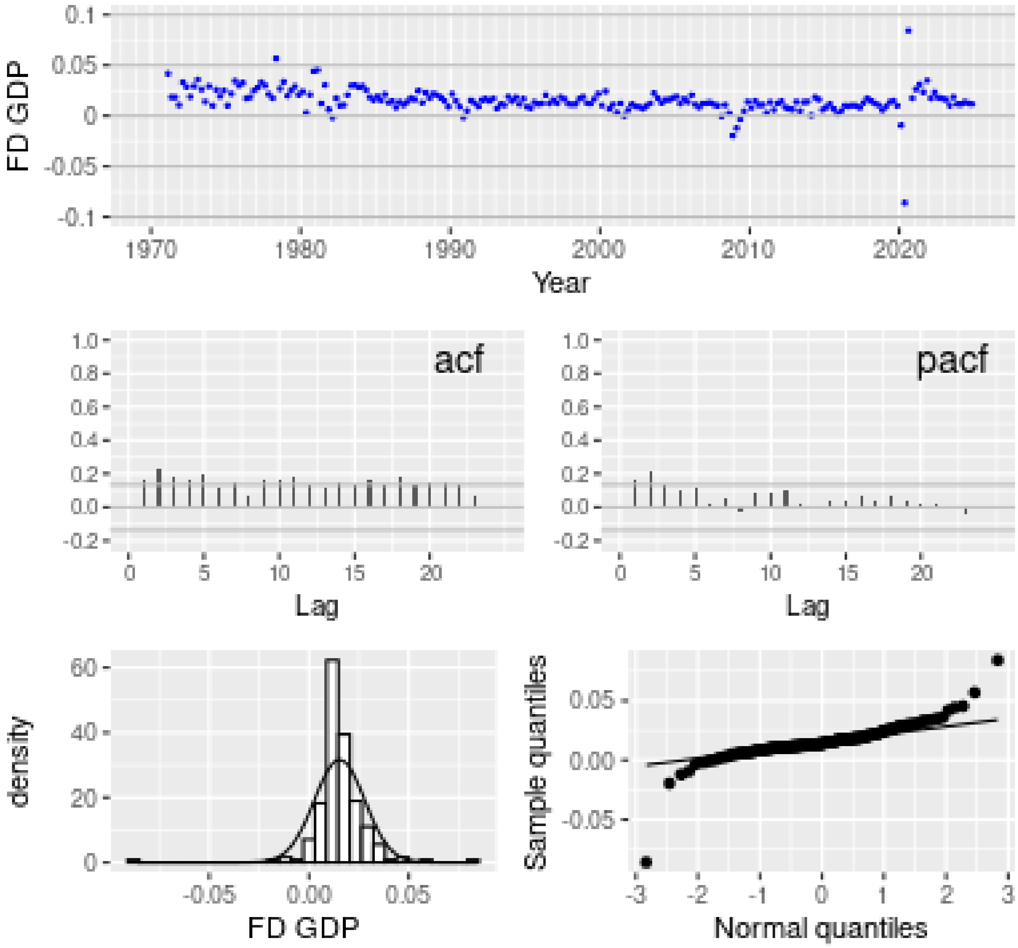

図 4　Time-series plot, correlogram, histogram, and QQ plot of first-differenced GDP

They are due to the coronavirus pandemic. The second row of Figure 4 is the correlogram of ΔlnY_t. The ACF is insignificant but persistent. The PACF is insignificant and damped out. The left panel of the third row is the histogram of ΔlnY_t. The right panel of the third row is the QQ plot of ΔlnY_t, and many scores follow closely the normal-distribution line. However, some scores at the lower end are far below the line, while some at the upper end are far above the line.

Table 4 shows the descriptive statistics and the hypothesis tests of first-differenced GDP, ΔlnY_t. Similarly to ΔlnS_t, the skewness is negative, and the kurtosis exceeds three. The ADF test rejects the null hypothesis of nonstationarity. In other words, first-differenced GDP is stationary. The Ljung-Box test rejects the null hypothesis of no autocorrelation. The Shapiro-Wilk test also rejects the null hypothesis of normality due to a negatively skewed and leptokurtic distribution. Thus, first-differenced GDP could be neither white nor Gaussian.

The upper panel of Figure 5 is the time-series plot of the first-differenced Buffett indicator, $\Delta lnBI_t$. The first-differenced Buffett indicator would not have any long-term trend, or could be constant over time. There are possibly four downward outliers and one upward outlier. The second row of Figure 5 is the correlogram of $\Delta lnBI_t$. The ACF is persistently significant and gradually damping out. The PACF of the first lag is statistically significant, but other lags are insignificant. In other words, the first-differenced Buffett indicator should be an AR(1) process, or the Buffett indicator should be an ARIMA(1, 1, 0) process. The left panel of the third row is the histogram of $\Delta lnBI_t$, which seems normal. The right panel of the third row is the QQ plot of $\Delta lnBI_t$. Most scores follow closely the normal-distribution line. However, there are possibly five outliers:

表 4　Descriptive statistics and hypothesis tests of first-diffenced GDP

Mean	Variance	Skewness	Kurtosis
0.0153104	1.5903612×10^{-4}	-1.4228093	25.4724515

Test	Test statistic	p value	H_0
ADF test	-4.97457	< 0.01	nonstationarity
Ljung-Box test	128.8635789 (df = 24)	2.220446×10^{-16}	no autocorrelation
Shapiro-Wilk test	0.7745882	$6.4347371 \times 10^{-17}$	normality

($T = 216$)

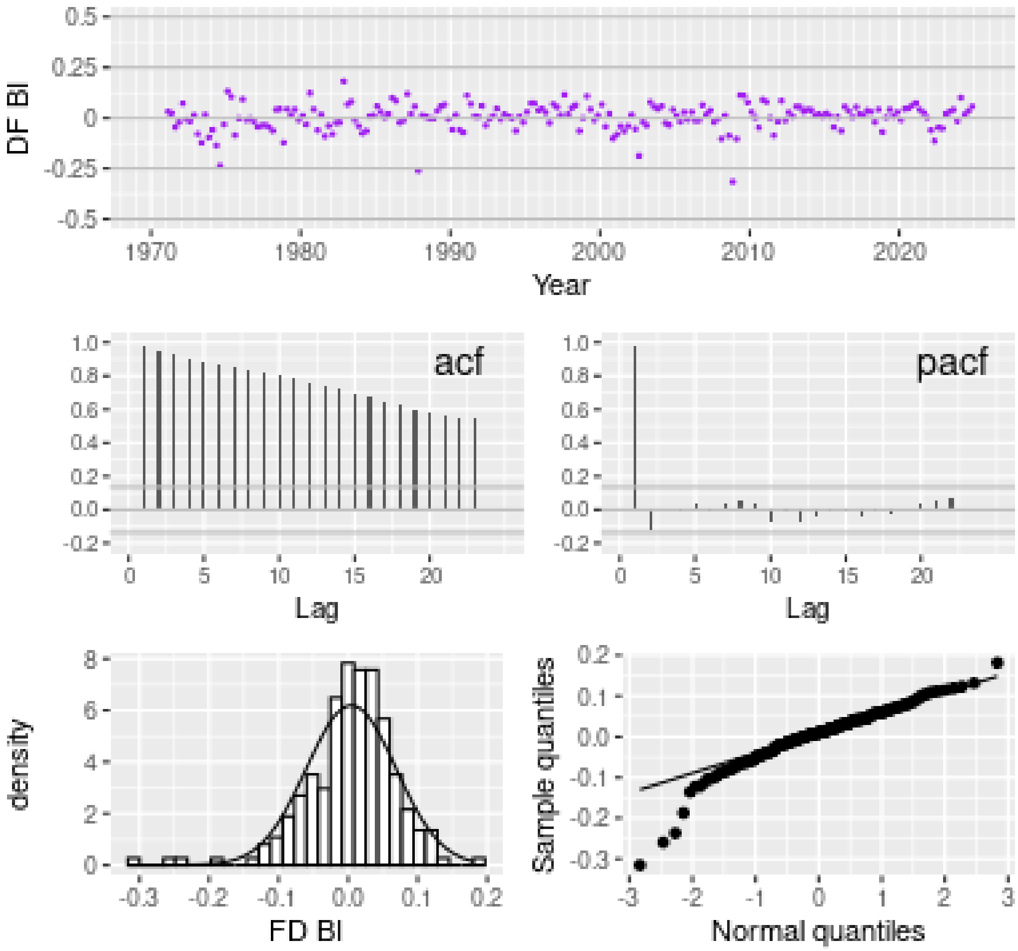

図 5　Time-series plot, correlogram, histogram, and QQ plot of first-differenced Buffett indicator

four at the lower end, and one at the upper end.

Table 5 shows the descriptive statistics and the hypothesis tests of the first-differenced Buffett indicator, $\Delta lnBI_t$. Similarly to ΔlnS_t and ΔlnY_t, the skewness is negative, and the kurtosis exceeds three. The ADF test rejects the null hypothesis of nonstationarity. In other words, the first-differenced Buffett indicator is stationary. The Ljung-Box test fails to reject the null hypothesis of no-autocorrelation at the 1% confidence level. The Shapiro-Wilk test rejects the null hypothesis of normality due to a negatively skewed and leptokurtic distribution. Thus, the first-differenced Buffett indicator could be white but not Gaussian.

The examined data, the Wilshire 5000 index (lnS_t), GDP (lnY_t), and the Buffett indicator ($lnBI_t$), are all

表 5 Descriptive statistics and hypothesis tests of first-diffenced Buffett indicator

(c) $\Delta lnBI_t$

Mean	Variance	Skewness	Kurtosis
0.0043711	0.0041579	-1.1512243	6.9527955

Test	Test statistic	p value	H_0
ADF test	-6.1531625	< 0.01	nonstationarity
Ljung-Box test	40.1335316 (df = 24)	0.0206913	no autocorrelation
Shapiro-Wilk test	0.936475	4.4110745×10^{-8}	normality

($T = 216$)

nonstationary time-series data. However, after first differencing, they become stationary. The histograms of the first-differenced data are negatively skewed and leptokurtic. Therefore, their distributions are not normal. Examining the first-differenced data suggests that there would be several outliers.

2 Ordinary Least Squares Analyses

This section conducts OLS estimations of three econometric models. Two models are simple linear regression models (SLRMs), and another is a multiple linear regression model, specifically the Error Correction Model (ECM). The first SLRM employs the data of lnS_t and lnY_t in their absolute forms, while the second LSRM employs the same data in their first-differenced form. Equations (2) and (3) are the econometric models of these two models. Equation(4) is the econometric model of the ECM for lnS_t and lnY_t, which employs both the absolute and the first-differenced forms of the data.

$$lnS_t = \alpha + \beta \cdot lnY_t + u_t \tag{2}$$
$$\Delta lnS_t = \gamma + \eta \cdot \Delta lnY_t + v_t \tag{3}$$
$$\Delta lnS_t = \phi + \psi \cdot \Delta lnY_t + \xi \cdot lnS_{t-1} + \zeta \cdot lnY_{t-1} + w_t \tag{4}$$

This section first conducts OLS estimations with all the data. They find that four residuals are outliers, so the study then conducts another OLS estimation with the data after removing outliers.

2.1 OLS analyses with all data

Table 6 shows coefficient estimates and hypothesis tests. Panel (a) of the Table is for SLRM with data in the absolute form. The estimated coefficient $\widehat{\beta}$ is 1.392, and rejects the null hypothesis, $\beta = 1$. This is compatible with the fact that the W5K has been increasing faster than GDP. The Durbin-Watson (DW) test rejects the null hypothesis of no autocorrelation. In addition, the DW test statistic is less than R^2, from which the rule of thumb suggests that the regression could be spurious.

Panel (b) of Table 6 is for SLRM with data in the first-differenced form. The estimated coefficient, $\widehat{\eta}$, is 1.072, and fails to reject the null hypothesis, $\eta = 1$. This is compatible with the Buffett indicator, which implies that the increasing rate of the W5K equals the GDP growth rate. The Durbin-Watson (DW) test rejects the null hypothesis of no autocorrelation. Similarly to the absolute form, the DW test statistic is less than R^2, from which the rule of thumb suggests spurious regression.

Panel (c) of Table 6 is for the ECM with data in the absolute and the first-differenced forms. The estimated coefficient, $\widehat{\psi}$, is 1.511, and fails to reject the null hypothesis, $\psi = 1$. This is compatible with the Buffet indicator,

表 6　Estimates and hypothesis tests of equation (2), (3), and (4)

(a) Estimates and hypothesis tests of SLRM equation (2)

Coefficient / test	Estimate / test statisitc	se	p value	Ho
α	-1.0489045	0.0468625	$4.4377642 \times 10^{-58}$	$= 0$
β	1.3924485	0.0212432	$4.3972163 \times 10^{-144}$	$= 0$
F test	4296.5166219	N/A	$4.3972163 \times 10^{-144}$	all coefficients equal zero
DW test	0.0532379	N/A	8.706969×10^{-48}	no autocorrelation

$T = 217$; $R^2 = 0.9523442$; $\overline{R^2} = 0.9521225$; SSR = 16.8651964

(b) Estimates and hypothesis tests of SLRM equation (3)

Coefficient / test	Estimate / test statisitc	se	p value	Ho
γ	0.0032698	0.0069259	0.6373299	$= 0$
η	1.0719352	0.3494945	0.0024403	$= 0$
F test	9.4071148	N/A	0.0024403	all coefficients equal zero
DW test	1.4616263	N/A	3.2997536×10^{-5}	no autocorrelation

$T = 216$; $R^2 = 0.0421075$; $\overline{R^2} = 0.0376314$; SSR = 0.8937759

(c) Estimates and hypothesis tests of ECM equation (4)

Coefficient / test	Estimate / test statisitc	se	p value	Ho
ϕ	-0.0679085	0.0229581	0.0034486	$= 0$
ψ	1.5114221	0.3759803	8.0895835×10^{-5}	$= 0$
ξ	-0.0393076	0.0156768	0.0129132	$= 0$
ζ	0.066251	0.0227001	0.0038963	$= 0$
F test	6.7904897	N/A	2.1579838×10^{-4}	all coefficients equal zero
DW test	1.4995011	N/A	6.2576215×10^{-5}	no autocorrelation

$T = 216$; $R^2 = 0.0876677$; $\overline{R^2} = 0.0747573$; SSR = 0.8512653

Relation	Estimate	se	p value	Ho
$\widehat{\zeta}/\widehat{\xi}$	-1.6854492	0.1730906	2.088677×10^{-22}	$= 0$

although its standard error is large. The ratio of estimates, $\widehat{\zeta}$ / $\widehat{\xi}$ is -1.686, whose absolute value is close to $\widehat{\psi}$. In other words, the long-term equilibrium level, $\widehat{\psi}$, and the short-term transition, $|\ \widehat{\zeta}\ /\ \widehat{\xi}\ |$, are approximately equal, or the Buffett indicator is valid for both the long-term and the short-term relation between lnS_t and lnY_t. The Durbin-Watson (DW) test rejects the null hypothesis of no autocorrelation. Similarly to the SLRM estimations, the DW test statistic is less than R^2, from which the rule of thumb suggests spurious regression.

Figure 6 shows the time-series plot, the correlogram, the histogram, and the QQ plot of SLRM residuals from the absolute-form regression, $\widehat{u}_t$. The time-series plot shows that the residuals $\widehat{u}_t$ are autocorrelated, and the correlogram agrees. The correlogram suggests that the residuals $\widehat{u}_t$ are an AR(1) process. Its histogram and QQ plot suggest that its distribution is not normal. Table 7 is the descriptive statistics and hypothesis tests for the residuals $\widehat{u}_t$. The p value of the Engel-Grange ADF test is between the critical values of 1% and 5%. The Engel-Grange test for cointegration employs the ADF test of regression residuals, but its critical values require adjustments due to estimated coefficients. The Ljung-Box test rejects the null hypothesis of no autocorrelation,

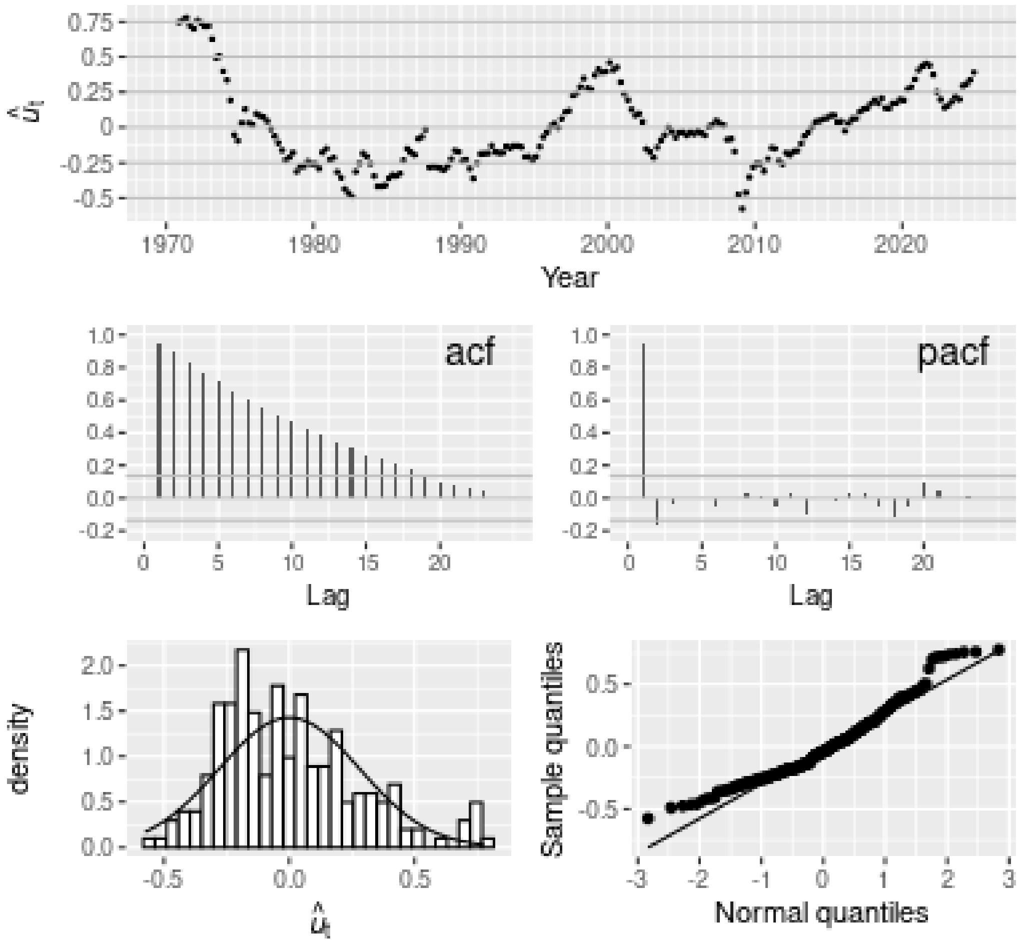

図 6 Time-series plot, correlogram, histogram, and QQ plot of SLRM residuals $\widehat{u}_t$

which is compatible with its correlogram. The Shapiro-Wilk test rejects the null hypothesis of normality due to a positively skewed distribution.

Figure 7 shows the time-series plot, the correlogram, the histogram, and the QQ plot of SLRM residuals from the first-differenced-form regression, $\widehat{v}_t$. The time-series plot shows that the residuals $\widehat{v}_t$ seem randomly scattered around zero, and the correlogram agrees. The correlogram suggests that the residuals $\widehat{v}_t$ is neither an AR(p) process, nor a MA(q) process. The histogram and QQ plot suggest five possible outliers: four at the lower end and one at the upper end. Table 8 is the descriptive statistics and hypothesis tests for the residuals $\widehat{v}_t$. The Engel-Grange ADF test rejects the null hypothesis of nonstationarity. The Ljung-Box test fails to reject the null hypothesis of no autocorrelation at the 1% confidence level. The Shapiro-Wilk test rejects the normality hypothesis due to a negatively skewed and leptokurtic distribution.

Figure 8 shows the time-series plot, the correlogram, the histogram, and the QQ plot of the ECM residuals, $\widehat{w}_t$. The time-series plot shows that the residuals $\widehat{w}_t$ seem randomly scattered around zero, and the correlogram agrees. The correlogram suggests that the residuals $\widehat{w}_t$ is neither an AR(p) process, nor a MA(q) process. Similarly to $\widehat{v}_t$, its histogram and QQ plot suggest that there are five possible outliers: four at the lower end and one at the upper end. Table 9 is the descriptive statistics and hypothesis tests for the residuals $\widehat{w}_t$. Similarly to $\widehat{v}_t$, the Engel-Granger ADF test rejects the null hypothesis of nonstationarity. The Ljung-Box test rejects the null hypothesis of no autocorrelation. The Shapiro-Wilk test also rejects the normality hypothesis due to a negatively skewed and leptokurtic distribution.

表 7　Descriptive statistics and hypothesis tests of SLRM residuals $\widehat{u}_t$

Mean	Variance	Skewness	Kurtosis
$8.6896056 \times 10^{-18}$	0.0780796	0.7508147	3.2697693

Test	Test statistic	p value	H_0
Engel-Granger ADF test	-3.5632537	$0.01 \sim 0.05$	nonstationarity
Ljung-Box test	1314.0820908 (df = 23)	0	no autocorrelation
Shapiro-Wilk test	0.9544044	2.175589×10^{-6}	normality

(T = 217)

Critical values for Engel-Granger ADF statistics

Number of regressors	0.10	0.05	0.01
1 (for $\widehat{u}_t$)	-3.12	-3.41	-3.96

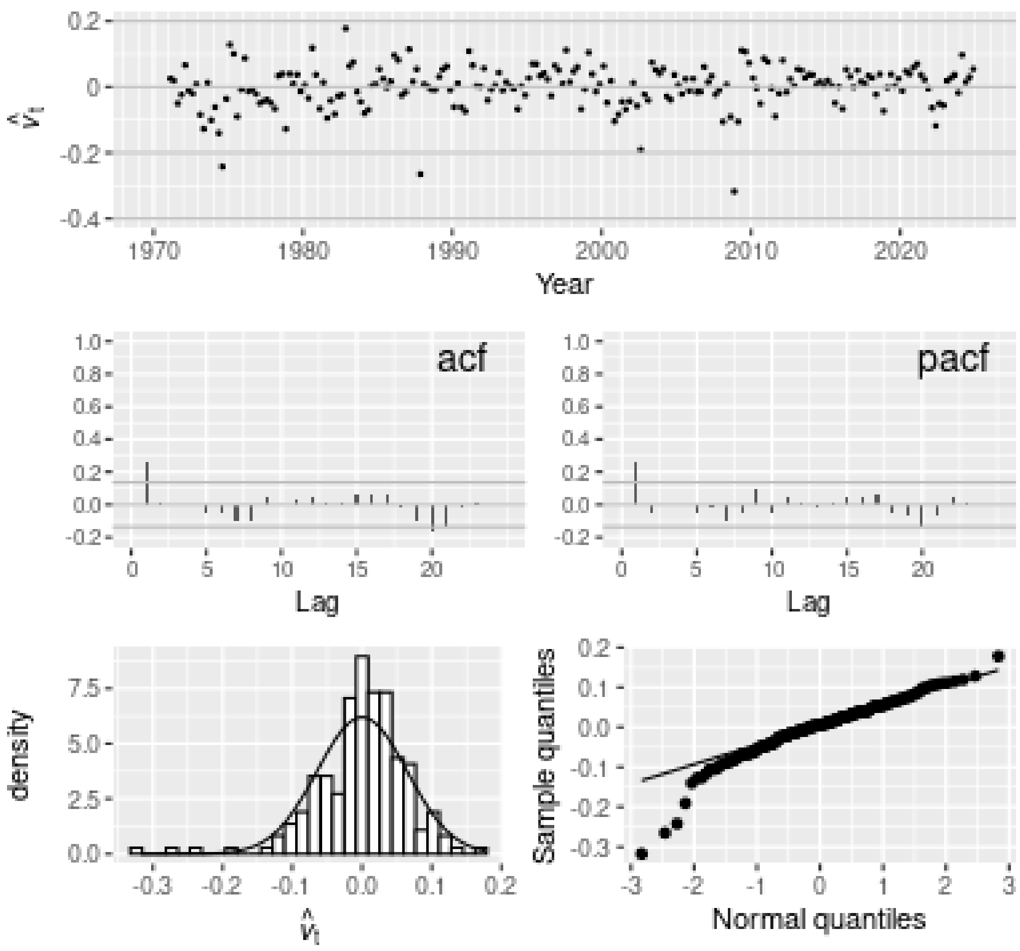

図 7　Time-series plot, correlogram, histogram, and QQ plot of SLRM residuals $\widehat{v}_t$

表 8　Descriptive statistics and hypothesis tests of SLRM residuals $\widehat{v}_t$

Mean	Variance	Skewness	Kurtosis
$-2.6667107 \times 10^{-18}$	0.0041571	-1.1406262	6.892945

Test	Test statistic	p value	H_0
Engel-Granger ADF test	-6.1460832	< 0.01	nonstationarity
Ljung-Box test	39.8431751 (df = 23)	0.0160028	no autocorrelation
Shapiro-Wilk test	0.9369766	4.8799484×10^{-8}	normality

($T = 216$)

Critical values for Engel-Granger ADF statistics

Number of regressors	0.10	0.05	0.01
1 (for $\widehat{v}_t$)	-3.12	-3.41	-3.96

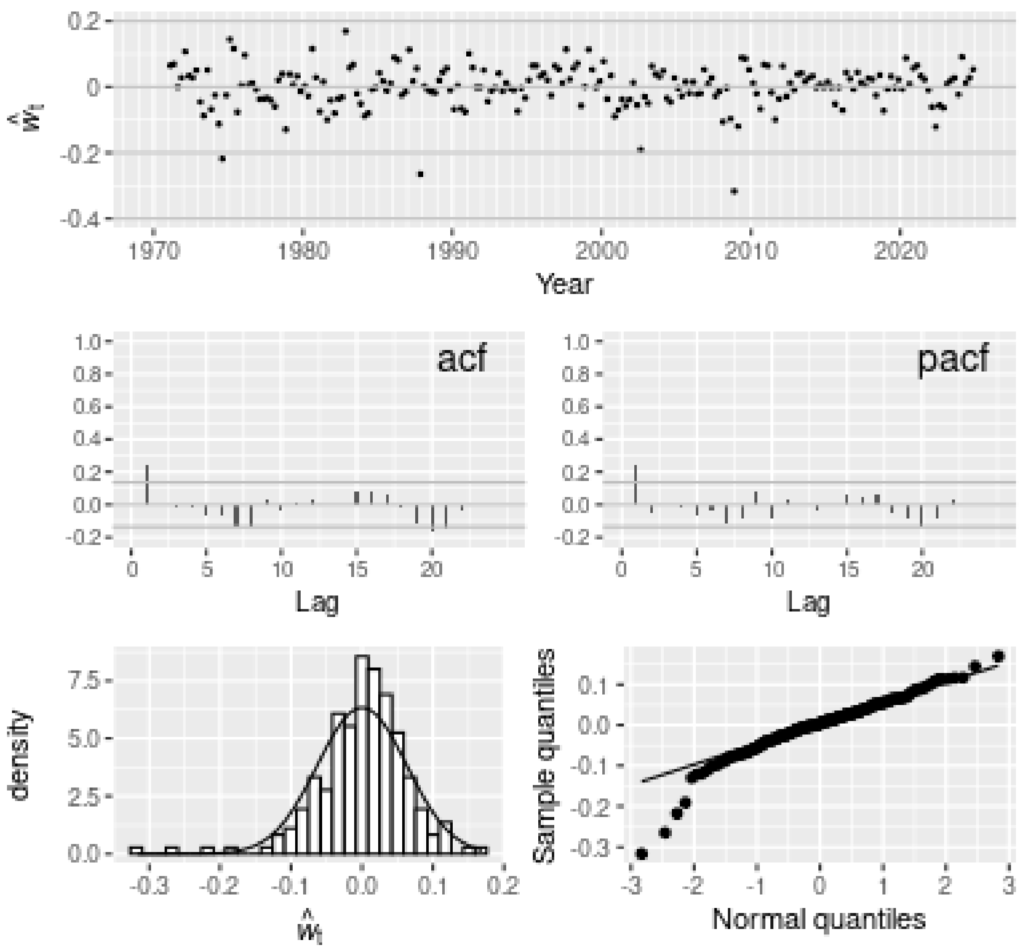

図 8　Time-series plot, correlogram, histogram, and QQ plot of ECM residuals $\widehat{w}_t$

表 9　Descriptive statistics and hypothesis tests of ECM residuals $\widehat{w}_t$

Mean	Variance	Skewness	Kurtosis
$4.7447649 \times 10^{-18}$	0.0039594	-1.0972483	7.071372

Test	Test statistic	p value	H_0
Engel-Granger ADF test	-6.3776455	< 0.01	nonstationarity
Ljung-Box test	43.3003337 (df = 21)	0.0028774	no autocorrelation
Shapiro-Wilk test	0.9390692	7.4762259×10^{-8}	normality

($T = 216$)

Critical values for Engel-Granger ADF statistics

Number of regressors	0.10	0.05	0.01
3 (for $\widehat{w}_t$)	-3.84	-4.16	-4.73

2.2 Outliers and OLS analyses without outliers

This study finds several possible outliers. Four exceed the three-sigma confidence interval, and all are negative. To accommodate the four outliers, the study splits the disturbance term of the ECM, w_t, into two elements, ordinary noise, w_t^*, and the outliers. The study regards the outliers as stochastic jumps. The studies further regards each jump as the product of a negative constant, η (< 0), and the Poisson process of with a rate λ, i.e., $\eta \cdot q_t$ where $q_t \sim Po(\lambda)$. Thus, $w_t = w_t^* + \eta \cdot q_t$. The estimates of η and λ are respectively ($\widehat{\eta} =$) -0.24736 and ($\widehat{\lambda} =$) 0.0184332, assuming $\eta \perp\!\!\!\perp q_t$.

Then, the study again estimates regression equations (2), (3), and (4) without the outliers, and superscript(*) indicates estimates and residuals from estimations without outliers. The study further examines whether the residuals are white Gaussian noise. When the Shapiro-Wilk test fails to reject the null hypothesis of normality, the study examines whether their mean and variance remain constant over the examined period by resorting to the t and the F tests for the equality of mean and variance. This is a test of whether the residuals are identically distributed. The study employs the Ljung-Box test to investigate whether the residuals are autocorrelated. This is a test of whether the residuals are independent.

2.2.1 Outliers and economic events

Table 10 shows the list of the four outliers. These outliers coincided with negative economic events. The first outlier occurred in the third quarter of 1974. In September 1973, Egypt waged a surprise attack on Israel. Israel, backed by the US, struck back. It is the 1973 Arab-Israeli Conflict. At the annual conference of the Organization of Petroleum Exporting Countries (OPEC) in October 1973, its Arab member countries declared an oil embargo against countries supporting Israel.[*14] In the following events, OPEC unilaterally raised the oil price. Although the embargo period was short, the high oil price persisted. The world economy plunged into a recession with high inflation, i.e., stagflation.

The second outlier happened in the fourth quarter of 1987. On Monday, October 19, 1987, the world stock markets crashed.[*15] Asian markets, except Japan, initiated the crash, and the crash spread westward. The Dow

[*14] *Wikipedia*, *The Free Encyclopedia*, s.v. "1973 oil crisis," (accessed January 6, 2025), `https://en.wikipedia.org/wiki/1973_oil_crisis`

[*15] *Wikipedia*, *The Free Encyclopedia*, s.v. "Black Monday (1987)," (accessed February 5, 2025), `https://en.wikipedia.org/wiki/Balck_Mondat_(1987)`

表 10　Four outliers

Year	Quarter	lnS_t	lnY_t	$\widehat{w}_t$	Economic event
1974	3	-0.484049	0.4447025	-0.2175046	1973 oil crisis
1987	4	0.9105538	1.6110354	-0.2644583	Black Monday
2002	3	2.1363207	2.3964433	-0.1909299	Dot-com crash
2008	4	2.2075261	2.6815836	-0.3165473	Lehman shock

Jones Industrial Average, for example, dropped by 508 points, or 22.6% in a single day, and it is the largest one-day drop in the index's history.

The third outlier was in the third quarter of 2002. In the second half of the 1990s, stock prices rose greatly, resulting in an economic bubble, i.e., the Dot-com bubble. One of the main factors was the widespread adoption of the multitasking MS Windows OS and the Internet. The stock price peaked in March 2000, and many communication service and online shopping firms have gone bankrupt.*[16] The quarter mean of the W5K in the third quarter of 1997 was $8.88 trillion, while its quarterly mean was $13.83 trillion at the peak of the Dot-com bubble in the third quarter of 2000. Its quarterly mean in the fourth quarter of 2002 was $8.37 trillion.

The fourth and last outlier occurred in the fourth quarter of 2008. On September 15, 2008, Lehman Brothers, one of the biggest investment banks in the world, went bankrupt, triggering the worldwide crash.*[17] The subprime mortgage crisis started in 2007, and its negative consequences irreparably undermined Lehman Brothers. Lehman Brothers was one of the first Wall Street firms to enter the mortgage business. The firm's high exposure to subprime-mortgage business resulted in its bankruptcy.

2.2.2 OLS analyses without outliers

This study finds four outliers in its dataset. Hereafter, the study conducts OLS estimations with data from which the outliers are removed. Table 11 shows the OLS estimates of econometric models (2), (3), and (4). The estimated coefficients of the SLRM equation (2) are similar to Table 6. The DW test statistic is far less than R^2, so the regression could be spurious. For the SLRM equation (3), the estimated coefficient $\widehat{\eta}$ is 0.700 down from 1.072 in Table 6. The DW test statistic is greater than R^2, so the regression could not be spurious. For the ECM equation (4), the estimated coefficient $\widehat{\psi}$ is 1.037, which is approximately equal to unity. The t test fails to reject the null hypothesis, $\psi = 1$. This is compatible with the Buffett indicator. The DW test statistic is greater than R^2, so the regression could not be spurious.

Figure 9 shows the time-series plot, the correlogram, the histogram, and the QQ plot of SLRM residuals from the absolute-form regression, $\widehat{u}_t^*$. The time-series plot shows that the residuals $\widehat{u}_t^*$ are autocorrelated, and the correlogram agrees. The correlogram suggests that the residuals $\widehat{u}_t$ are an AR(1) process. Its histogram and QQ plot suggest that its distribution is not normal. Table 12 is the descriptive statistics and hypothesis tests for the residuals $\widehat{u}_t^*$. The Ljung-Box test rejects the null hypothesis of no autocorrelation, which is compatible with its correlogram. The Shapiro-Wilk test rejects the normality hypothesis.

Figure 10 shows the time-series plot, the correlogram, the histogram, and the QQ plot of SLRM residuals from the first-differenced-form regression, $\widehat{v}_t^*$. The time-series plot shows that the residuals $\widehat{v}_t^*$ seem randomly scattered around zero, and the correlogram agrees. The correlogram suggests that the residuals $\widehat{u}_t^*$ is neither an AR(p) process, nor a MA(q) process. The histogram and QQ plot suggest no outliers; all scores lie along the normal-distribution line. Table 13 is the descriptive statistics and hypothesis tests for the residuals $\widehat{v}_t^*$.

*[16] *Wikipedia*, *The Free Encyclopedia*, s.v. "Dot-com bubble," (accessed January 6, 2025), `https://en.wikipedia.org/wiki/Dot-com_bubble`

*[17] *Wikipedia*, *The Free Encyclopedia*, s.v. "Bankruptcy of Lehman Brothers," (accessed January 6, 2025), `https://en.wikipedia.org/wiki/Bankruptcy_of_Lehman_Brothers`

表 11　Estimates and hypothesis tests of equations (2), (3), and (4) without outliers

(a) Estimates and hypothesis tests of SLRM equation (2) without outliers

Coefficient / test	Estimate / test statisitc	se	p value	Ho
α^*	-1.0463842	0.0473523	$8.2271295 \times 10^{-57}$	$= 0$
β^*	1.393449	0.0214267	$1.4426862 \times 10^{-141}$	$= 0$
F test	4229.3370722	N/A	$1.4426862 \times 10^{-141}$	all coefficients equal zero
DW test	0.0594985	N/A	$1.1305235 \times 10^{-46}$	no autocorrelation

$T = 213$; $R^2 = 0.9524811$; $\overline{R^2} = 0.9522559$

(b) Estimates and hypothesis tests of SLRM equation (3) without outliers

Coefficient / test	Estimate / test statisitc	se	p value	Ho
γ^*	0.0146927	0.0060362	0.0157809	$= 0$
η^*	0.700323	0.3033399	0.0219517	$= 0$
F test	5.3301289	N/A	0.0219517	all coefficients equal zero
DW test	1.609438	N/A	0.0022459	no autocorrelation

$T = 208$; $R^2 = 0.0252218$; $\overline{R^2} = 0.0204899$

(c) Estimates and hypothesis tests of ECM equation (4) without outliers

Coefficient / test	Estimate / test statisitc	se	p value	Ho
ϕ^*	-0.0505622	0.0193246	0.0095491	$= 0$
ψ^*	1.0372751	0.3220558	0.0014878	$= 0$
ξ^*	-0.0413525	0.0131816	0.0019583	$= 0$
ζ^*	0.0658714	0.0190967	6.8267414×10^{-4}	$= 0$
F test	6.1295061	N/A	5.1954884×10^{-4}	all coefficients equal zero
DW test	1.6298862	N/A	0.002392	no autocorrelation

$T = 208$; $R^2 = 0.0826865$; $\overline{R^2} = 0.0691966$

Relation	Estimate	se	p value	Ho
$\widehat{\zeta}^*/\widehat{\xi}^*$	-1.5929226	0.1221724	7.407342×10^{-39}	$= 0$
$-\widehat{\phi}^*/\widehat{\xi}^*$	-1.2227102	0.2995513	4.4687186×10^{-5}	$= 0$
$-(\widehat{\psi}^* + \widehat{\zeta}^*/\widehat{\xi}^*)$	0.5556475	0.3124617	0.0753563	$= 0$

表 12　Descriptive statistics and hypothesis tests of SLRM residuals $\widehat{u}_t^*$

Mean	Variance	Skewness	Kurtosis
$-5.7045577 \times 10^{-17}$	0.0779531	0.7538038	3.2647381

Test	Test statistic	p value	H_0
Engel-Granger ADF test	N/A due to omission of outliers		nonstationarity
Ljung-Box test	1336.87498 (df = 23)	0	no autocorrelation
Shapiro-Wilk test	0.9536869	2.2365047×10^{-6}	normality

($T = 213$)

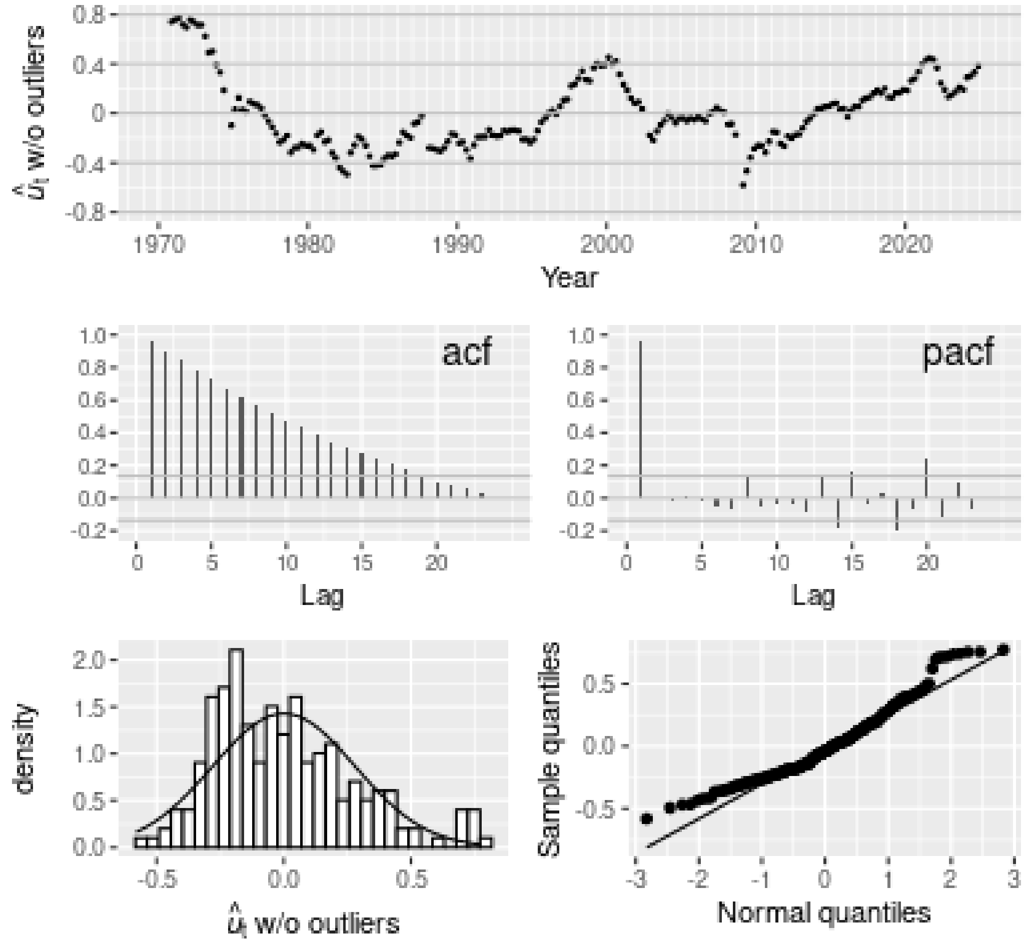

図 9 Time-series plot, correlogram, histogram, and QQ plot of SLRM residuals $\widehat{u}_t^*$

表 13 Descriptive statistics and hypothesis tests of SLRM residuals $\widehat{v}_t^*$

(b) SLRM residuals without outliers ($\widehat{v}_t^*$)

Mean	Variance	Skewness	Kurtosis
$2.2333001 \times 10^{-18}$	0.0029185	-0.0851944	3.0398462

Test	Test statistic	p value	H_0
Engel-Granger ADF test	N/A due to omission of outliers		nonstationarity
Ljung-Box test	46.8273791 (df = 23)	0.0023567	no autocorrelation
Shapiro-Wilk test	0.9949904	0.7232631	normality

($T = 208$)

The Ljung-Box test rejects the null hypothesis of no autocorrelation. The Shapiro-Wilk test fails to reject the normality hypothesis.

Figure 11 shows the time-series plot, the correlogram, the histogram, and the QQ plot of the ECM residuals, $\widehat{w}_t^*$. The time-series plot shows that the residuals $\widehat{w}_t^*$ seem randomly scattered around zero, and the correlogram agrees. The correlogram suggests that the residuals $\widehat{w}_t^*$ is neither an AR(p) process, nor a MA(q) process. Similarly to the residuals $\widehat{v}_t^*$, its histogram and QQ plot suggest that there is no outlier, and all scores lie along the normal-distribution line. Table 14 is the descriptive statistics and hypothesis tests for the residuals $\widehat{w}_t^*$.

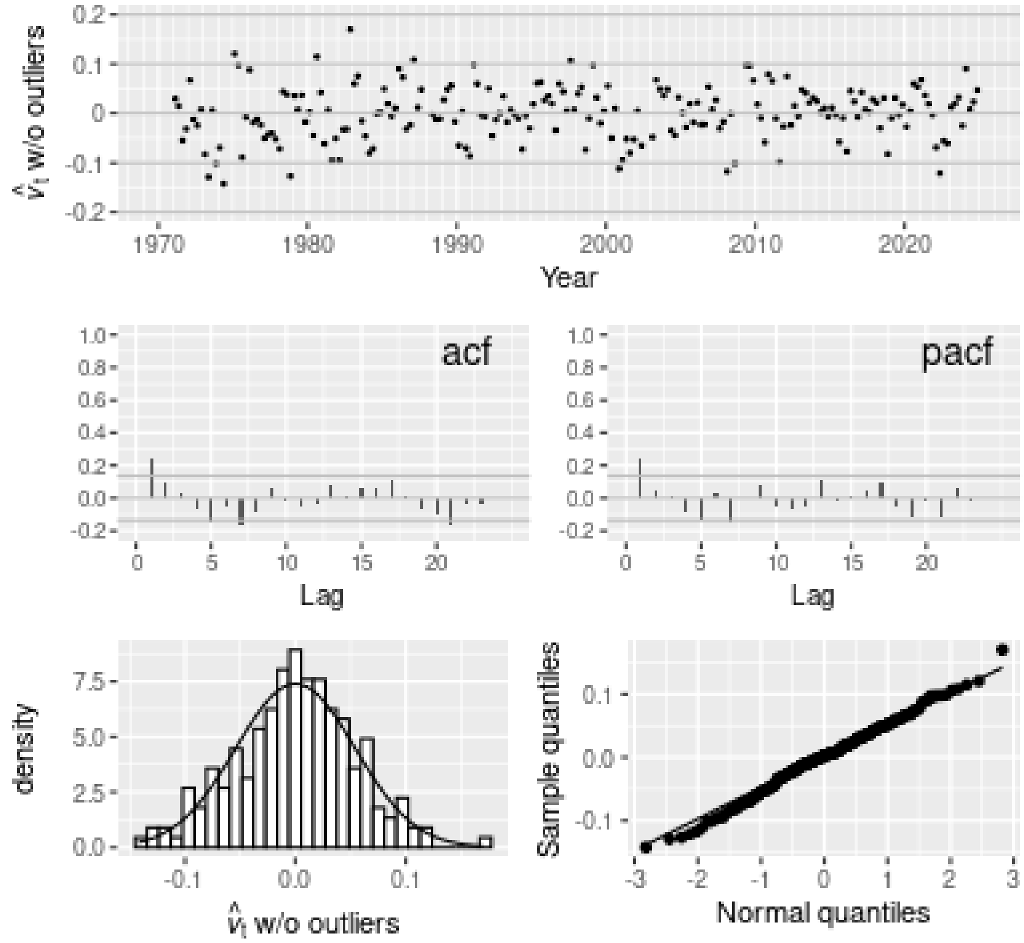

図 10　Time-series plot, correlogram, histogram, and QQ plot of SLRM residuals $\widehat{v}_t^*$

表 14　Descriptive statistics and hypothesis tests of ECM residuals $\widehat{w}_t^*$

Mean	Variance	Skewness	Kurtosis
$-1.1567864 \times 10^{-18}$	0.0027465	-0.0185119	2.9655779

Test	Test statistic	p value	H_0
Engel-Granger ADF test	N/A due to omission of outliers		nonstationarity
Ljung-Box test	52.8414896 (df = 21)	1.4490329×10^{-4}	no autocorrelation
Shapiro-Wilk test	0.9954866	0.79759	normality

(T = 208)

The Ljung-Box test rejects the null hypothesis of no autocorrelation. The Shapiro-Wilk test fails to reject the normality hypothesis.

For comparison, the study examines the first-differenced W5K without outliers, ΔlnS_t^*. Figure 12 shows the time-series plot, the correlogram, the histogram, and the QQ plot of the first-differenced W5K, ΔlnS_t^*. The time-series plot shows that ΔlnS_t^* seems randomly scattered around zero, and the correlogram agrees. The correlogram suggests that ΔlnS_t^* is neither an AR(p) process, nor a MA(q) process. Similarly to $\widehat{v}_t^*$ and $\widehat{w}_t^*$, its histogram and QQ plot suggest that there is no outlier, and all scores lie along the normal-distribution line. Table 15 is the descriptive statistics and hypothesis tests for ΔlnS_t^*. The Ljung-Box test rejects the null

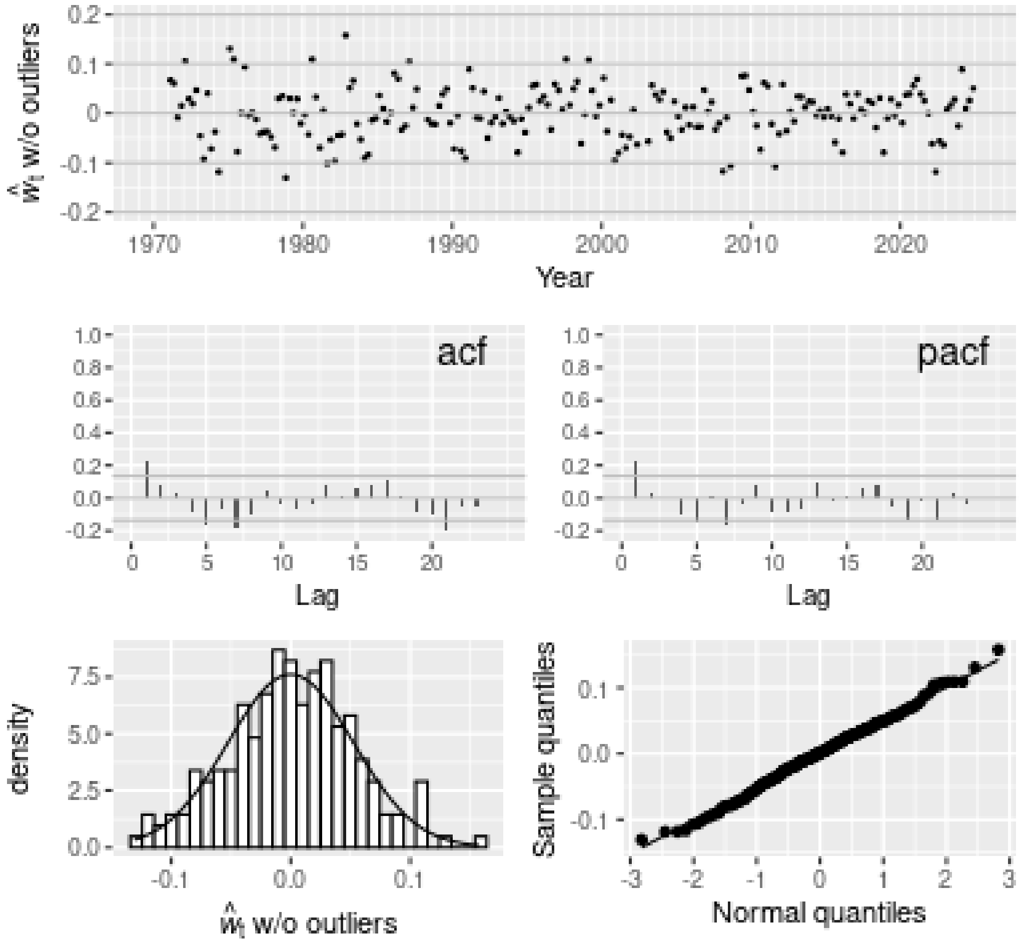

図 11　Time-series plot, correlogram, histogram, and QQ plot of ECM residuals $\widehat{w}_t^*$

表 15　Descriptive statistics and hypothesis tests of first-differenced W5K ΔlnS_t^*

Mean	Variance	Skewness	Kurtosis
0.0256039	0.002994	-0.0585525	2.9139929

Test	Test statistic	p value	H_0
ADF test	N/A due to omission of outliers		nonstationarity
Ljung-Box test	48.231377 (df = 24)	0.0023628	no autocorrelation
Shapiro-Wilk test	0.99497	0.7201196	normality

(T = 216)

hypothesis of no autocorrelation. The Shapiro-Wilk test fails to reject the normality hypothesis.

Without outliers, the ECM residuals, $\widehat{w}_t^*$, and the first-differenced W5K, ΔlnS_t, are likely not only stationary but also Gaussian. The study further examines whether they are identically distributed. The normal distribution is a two-parameter distribution; the two parameters are the mean and the variance. The study splits its time-series data into two periods: the first half and the second half. Firstly, the study employs the F test to compare the variances, and its null hypothesis is that the two variances are equal. The last row of Table 16 shows the results. The F tests fail to reject the null hypothesis at the 1% confidence level. Then, the study conducts the t test for comparing the means, assuming equal variance. Its null hypothesis is that the means are equal. The

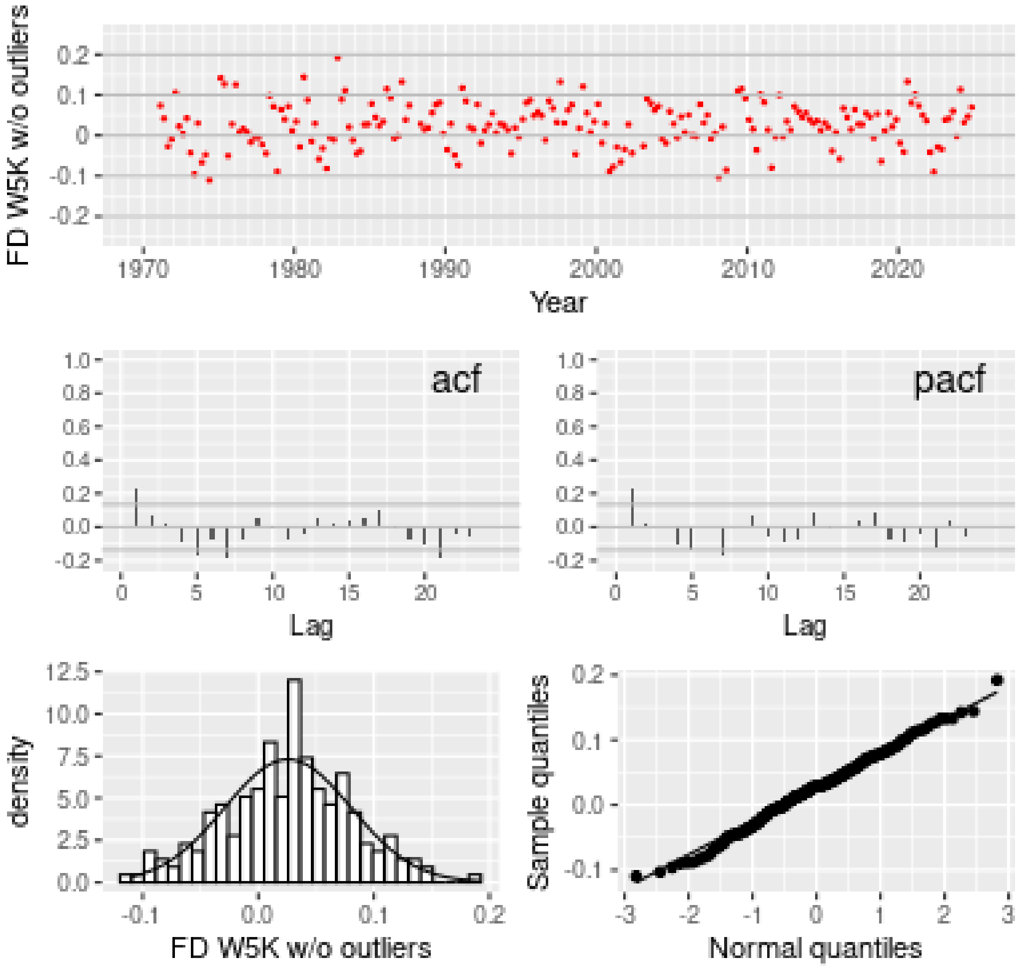

図 12　Time-series plot, correlogram, histogram, and QQ plot of first-differenced W5K ΔlnS_t^*

表 16　Hypothesis tests of comparing means and variances

Data	ECM residuals without outliers ($\widehat{w}_t^*$)		First-differenced W5K (ΔlnS_t)		
	Test statistic	p value	Test statistic	p value	Ho
t tests for means	0.2448419	0.8068226	0.5663242	0.5717668	equality
F tests for variances	1.4754848	0.0494446	1.0309861	0.8748846	equality

first row of Table 16 shows the results. The t tests fail to reject the null hypothesis. Therefore, $\widehat{w}_t^*$ and ΔlnS_t are likely identically distributed.

2.2.3 Buffett indicator as mean-reverting process

This study defines the empirical Buffett indicator, $lnBI_t^{[e]}$ as $lnS_t - \psi \cdot lnY_t$, i.e., $lnBI_t^{[e]} \equiv lnS_t - \psi \cdot lnY_t$, with $\psi \approx 1$. By rewriting the ECM of equation (4), we have the following:

$$(\Delta lnS_t - \psi \cdot \Delta lnY_t) = \phi + (\psi \cdot \xi + \zeta) \cdot lnY_{t-1} + \xi \cdot (lnS_{t-1} - \psi \cdot lnY_{t-1}) + w_t^* + \eta \cdot q_t$$

$$\Delta lnBI_t^{[e]} = \phi + (\psi \cdot \xi + \zeta) \cdot lnY_{t-1} + \xi \cdot lnBI_{t-1}^{[e]} + w_t^* + \eta \cdot q_t \tag{5}$$

$$= \kappa \cdot \left(\mu + \nu \cdot lnY_{t-1} - lnBI_{t-1}^{[e]}\right) + w_t^* + \eta \cdot q_t \tag{6}$$

where $\kappa \equiv -\xi$, $\mu \equiv -\phi/\xi$, and $\nu \equiv -(\psi + \zeta/\xi)$. Their estimates are respectively, ($\widehat{\kappa}^* =$) 0.0413525, ($\widehat{\mu}^* =$) -1.2227102, ($\widehat{\nu}^* =$) 0.5556475. Because the estimated coefficient $\widehat{\xi}^*$ is negative and greater than -1 so that

$0 < 1+\widehat{\xi}^*$ (< 1) or $0 < \widehat{\kappa}^*$ (< 1), the ECM of equation (4) is a mean-reverting process, or the empirical Buffett indicator likely shows regression to the mean. Therefore, when the empirical Buffett indicator is greater or less than the reversion mean, the indicator tends to decrease or increase, accordingly. The half-life of the process is 16.7619082 quarters.

Here, the reversion mean for the empirical Buffett indicator at time t is written as $\mu + \nu \cdot lnY_t$, which is a function of GDP. The estimated coefficient $\widehat{\nu}$ is positive, so the empirical Buffett indicator tends to increase along with GDP.

3 Expectations and simulations of empirical Buffett indicator and W5K

This section first calculates the expectations of the empirical Buffett indicator, $lnBI_t^{[e]}$, and the W5K, lnS_t, conditional on GDP, lnY_t. Then, the section simulates the empirical Buffett indicator and the W5K with GDP. The section employs two initial values for calculating the expectations and the simulations. The first one is the first GDP data in the first quarter of 1947. The second one is the first data of the W5K in the fourth quarter of 1970. The study calls the former "long," and the latter "short." For the time index, the long operations use Roman numerals, and the short operations use Hindu-Arabic numerals. Quarter 4 of the year 1970 is t = XCVI for the long operations, and $t = 1$ for the short operations.

By rewriting equation (5), we can write the expected value of the empirical Buffett indicator conditional on GDP as follows:

$$E[lnBI_t^{[e]} \,|\, lnY\text{'s}] = \phi + (\psi \cdot \xi + \zeta) \cdot lnY_{t-1} + (1+\xi) \cdot E[lnBI_{t-1}^{[e]} \,|\, lnY\text{'s}] + \eta \cdot \lambda \tag{7}$$

Then, we can write the conditional expectation of the W5K as follows:

$$E[lnS_t \,|\, lnY\text{'s}] = E[lnBI_t^{[e]} \,|\, lnY\text{'s}] + \psi \cdot lnY_t \tag{8}$$

The set of the parameters in equation (7), $\omega(\equiv \phi, \psi, \xi$ and $\zeta)$, employs the OLS estimates of the ECM without outliers, $\widehat{\omega}^*$. For the long operations, the initial value for the empirical Buffett indicator is the reversion mean at time I, i.e., $lnBI_{\text{I}}^{[e]} = \widehat{\mu}^* + \widehat{\nu}^* \cdot lnY_{\text{I}}$. The values of the coefficients $\widehat{\mu}^*$ and $\widehat{\nu}^*$ are the OLS-ECM estimates without outliers. For the short operations, the initial value for the empirical Buffett indicator is the observed empirical Buffett indicator at time 1, i.e., $lnBI_1^{[e]} = lnS_1 - \widehat{\psi}^* \cdot lnY_1$. The study calculates the expectations of the Buffett indicator and the W5K conditional on GDP. The green and turquoise curves show the long and short expectations in figures 13 and 14.

Figures 13 and 14 respectively show the simulations of the empirical Buffett indicator and the W5K of the long and the short simulations. The simulation parameters, $\widetilde{\omega}$, are random draws from the 4-variate normal distribution whose mean and veriance-covariance are respectively the OLS-ECM estimates without outliers and their variance-covariance matrix divided by the square root of T, i.e., $\widetilde{\omega} \sim N_{[4]}(\widehat{\omega}^*,\ \widehat{\Sigma}_{\widehat{\omega}^*}/\sqrt{T})$. The division of the variance reflects the superconsistency of cointegration. The simulated values of the error term w_t^* are random draws from the normal distribution with mean zero and variance $Var(\widehat{w}_t^*)$, i.e., $\widetilde{w}_t^* \sim N(0, Var(\widehat{w}_t^*))$. The value of another simulation parameter η equals the mean of the four outliers, i.e., $\widehat{\eta}$ = -0.24736. The simulated values of the other error term q_t are random draws from the Poisson distribution with parameter λ. Here, $\widehat{\lambda} = 0.0184332$.

Violet points in the upper panels of figures 13 and 14 are the first 1000 plots of the simulated empirical Buffett indicator. Then, for each quarter, the study calculates the mean (black solid line), the median (yellow dotted line), and the 95% confidence interval (yellow dotted lines). The corresponding lines almost overlap because the mean and the median are close. The observed empirical Buffett indicator has been fluctuating around the mean and the median of the simulated empirical Buffett indicator. Also, most points of the observed empirical

表 17　Data and descriptive statistics of long simulations

(a) Simulated W5K

Year	Quarter	Observed W5K (Trillion $)	Conditional mean (Trillion $)
2024	4	58.2772161	49.0423784

Remark: Observed GDP (Trillion $) : 29.723864

Percentiles of simulated W5K

(Mean)	(SD)	Median	(Precentile of observed W5K)
(49.188557)	(1.2490358)	49.6007127	(77.59 percentile)

0.5 percentile	2.5 percentile	5.0 percentile	95.0 percentile	97.5 percentile	99.5 percentile
26.3863332	31.2685133	33.7885066	70.0139693	74.9664681	85.4036484

(b) Simulated empirical Buffett indicator

Year	Quarter	Observed empirical Buffett indicator	Conditional mean
2024	4	1.9611606	1.4539721

Percentiles of simulated empirical Buffett indicator

(Mean)	(SD)	Median	(Precentile of observed empirical Buffett indicator)
(1.4549574)	(1.430242)	1.4599014	(79.87 percentile)

0.5 percentile	2.5 percentile	5.0 percentile	95.0 percentile	97.5 percentile	99.5 percentile
0.5653607	0.7133131	0.8018119	2.6071125	2.9324362	3.5879749

(Number of simulations = 2×10^4, execution time = 51.4613136927287 secs)

Buffett indicator remain within the 95% confidence interval. The starting point of the observed empirical Buffett indicator in 1970 was high relative to the reversion mean. Still, the mean and the median of the simulated empirical Buffett indicator reached the reversion mean after about fifteen years.

Pink points in the lower panels of figures 13 and 14 are the simulated W5K. Similarly to the simulated empirical Buffett indicator, for each quarter, the study calculates the mean (black solid line), the median (magenta dotted line), and the 95% confidence interval (magenta dotted lines). The corresponding lines almost overlap because the mean and the median are close. The observed W5K has been fluctuating around the mean and the median of the simulated W5K. Most points of the W5K remain within the 95% confidence interval. The starting point of the observed W5K in 1970 was high relative to the reversion mean, but the mean and the median of the simulated W5K reached the reversion mean after about fifteen years.

Figure 15 shows the histograms of the simulated empirical Buffett indicator and the W5K in quarter 1 of 2025. The upper panels are the long simulations, and the lower panels are the short simulations. Tables 17 and 18 demonstrate the descriptive statistics at the end of the long and short simulations.

By rewriting equation (5), we have the expected future value of the empirical Buffett indicator, conditional on the current value.

$$E[lnBI^{[e]}_{T+1} \,|\, lnY_T,\, lnBI^{[e]}_T] = \phi + (\psi \cdot \xi + \zeta) \cdot lnY_T + (1+\xi) \cdot lnBI^{[e]}_T + \eta \cdot \lambda \tag{9}$$

This is the "now"cast of the empirical Buffett indicator. At the end of each quarter, the W5K data is immediately available, while it takes some time for the GDP data to be published. Therefore, the current value

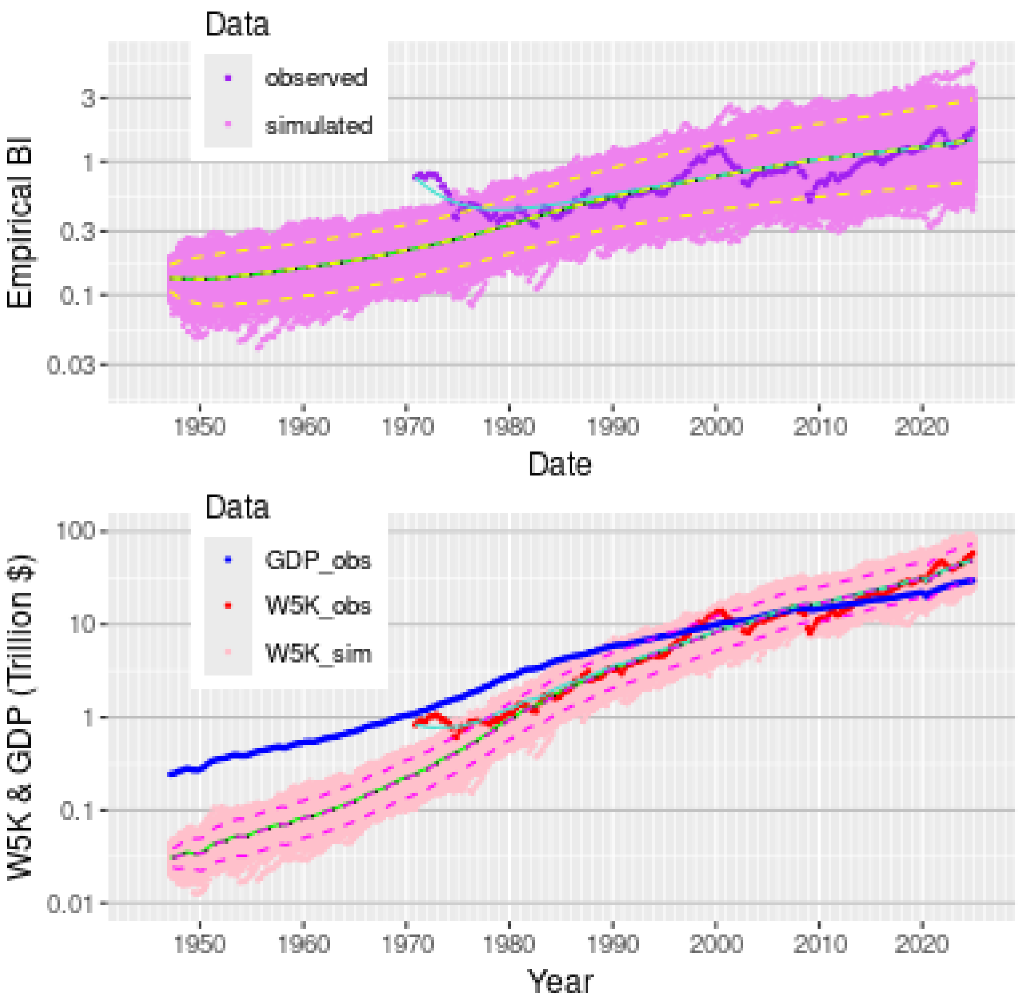

図 13 Long simulations of empirical Buffett indicator and W5K

of the empirical Buffett indicator is also unavailable. However, with the coefficient estimates, $\widehat{\omega}^*$, $\widehat{\eta}$ and $\widehat{\lambda}$, equation (9) provides the conditional expectation or the nowcast of the empirical Buffett indicator. For quarter 1 of the year 2025, the nowcast of the empirical Buffett indicator is 1.728112. Then, the nowcast of GDP is 29.7180222 trillion $. The observed values of the empirical Buffett indicator and GDP in quarter 4 of 2024 are 1.7277597 and 29.723864 trillion $.

4 Conclusions

This study investigates the dynamics of the Buffett indicator, which is the ratio of the stock price index, the Wilshire 5000 index, and GDP. The study confirms that the Wilshire 5000 and GDP are both nonstationary as they are known, but that they are cointegrated. The Buffett indicator is a mean-reverting Gaussian process

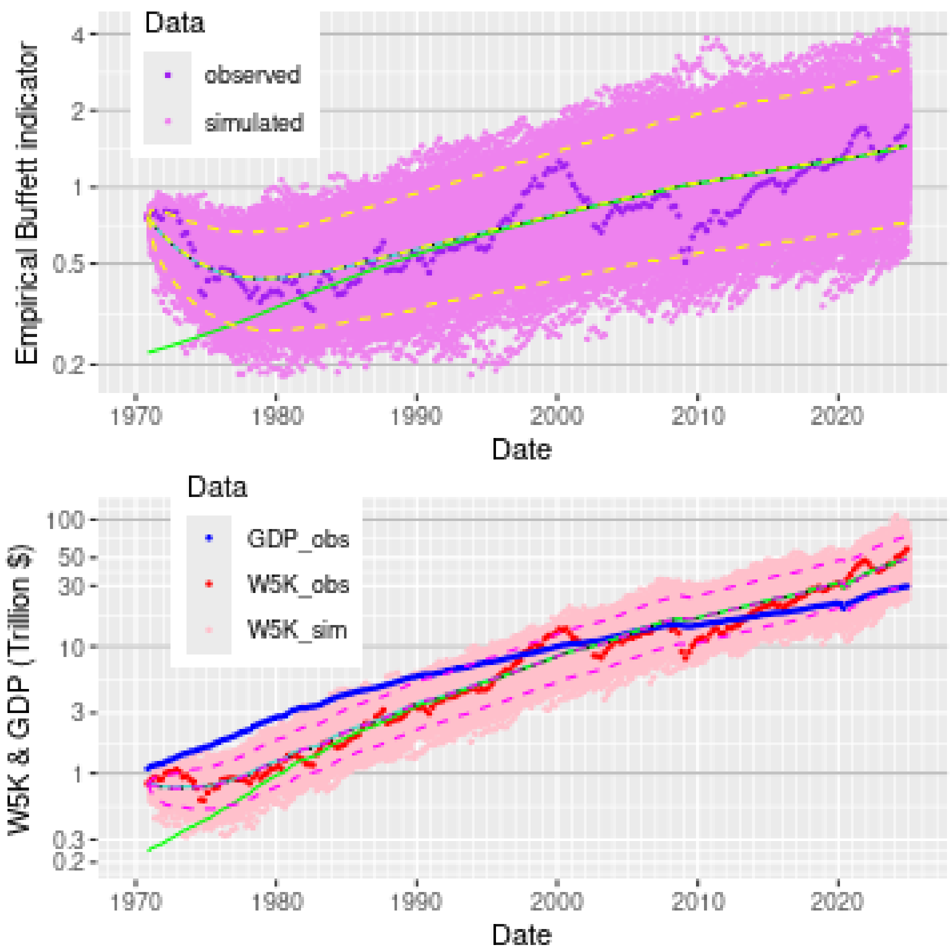

図 14　Short simulations of empirical Buffett indicator and W5K

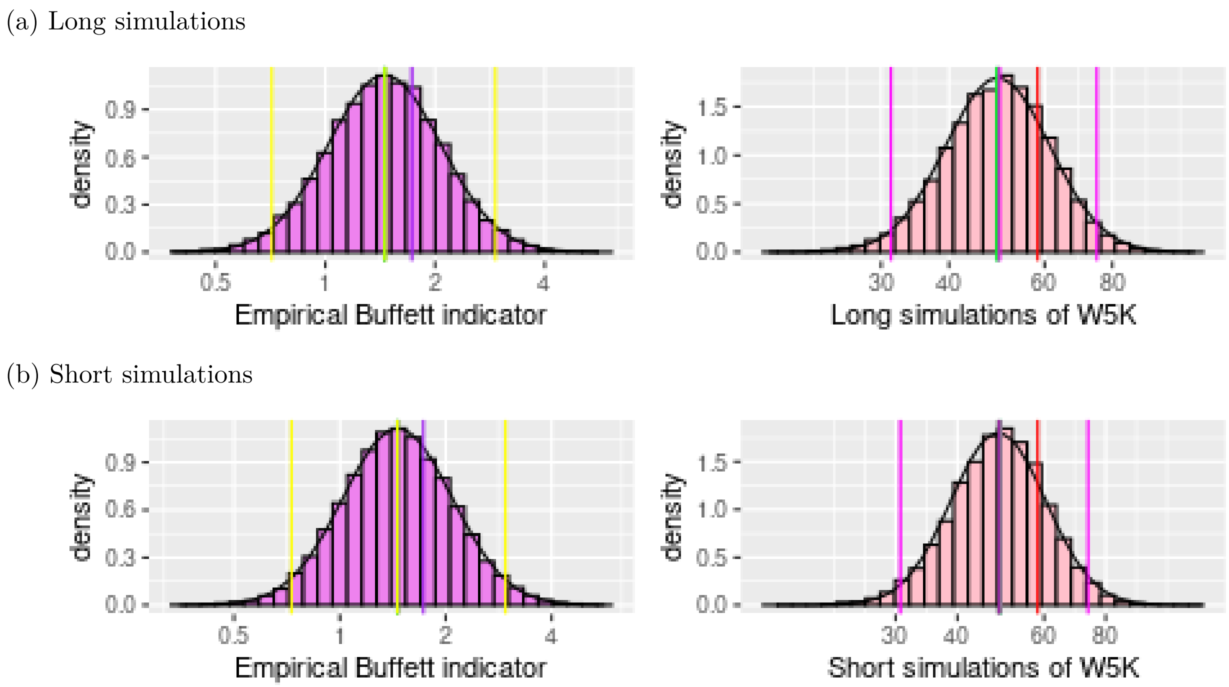

図 15　Histograms of simulations

表 18 Data and descriptive statistics of short simulations

(a) Simulated W5K

Year	Quarter	Observed W5K (Trillion \$)	Conditional mean (Trillion \$)
2024	4	58.2772161	49.0489442

Remark: Observed GDP (Trillion \$) : 29.723864

Percentiles of simulated W5K

(Mean)	(SD)	Median	(Precentile of observed W5K)
(48.4520586)	(1.2486384)	48.8354362	(80.06 percentile)

0.5 percentile	2.5 percentile	5.0 percentile	95.0 percentile	97.5 percentile	99.5 percentile
25.6440861	30.6398634	33.1603424	68.8463835	73.6462668	83.403095

(b) Simulated empirical Buffett indicator

Year	Quarter	Observed empirical Buffett indicator	Conditional mean
2024	4	1.9611606	1.4541668

Percentiles of simulated empirical Buffett indicator

(Mean)	(SD)	Median	(Precentile of observed empirical Buffett indicator)
(1.4583864)	(1.4318387)	1.4565679	(79.48 percentile)

0.5 percentile	2.5 percentile	5.0 percentile	95.0 percentile	97.5 percentile	99.5 percentile
0.5634677	0.7275781	0.8107452	2.6352283	2.9537563	3.7040691

(Number of simulations = 2×10^4, execution time = 25.5748508771261 secs)

with a Poisson jump process. The reversion mean of the Buffett indicator is increasing along with GDP, as is often pointed out. However, the increasing rate of the Wilshire 5000 equals the DGP growth rate, as the Buffett indicator implies. The study detects four outliers in the Wilshire 5000, which coincided with four adverse shocks to the US economy. The error correction model of the Buffett indicator is variance stationary as opposed to the efficient-market hypothesis.

5 Future research issues

There are several research issues. One is the variance of the OLS-ECM estimates. This study assumes the estimates are super consistent. However, the OLS-ECM estimates could be consistent rather than super consistent. Another one is a more extended dataset of the stock price index. A candidate could be OECD data.

6 References

- Buffett, Warren, and Carol Loomis, ”Warren Buffett on the Stock Market,” the *Fortune* Magazine, December 2001.
- Stock, James H., ”Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors,” *Econometrica*, September 1987.